Prony-type polynomials and their common zeros

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The problem of parameter estimation of an exponential sum $f(\mathbf{k}) = \sum_{j=1}^{N} a_j \exp\left(-\mathrm{i}\langle \boldsymbol{\omega}_j, \mathbf{k}\rangle\right)$, where $a_1, \ldots, a_N \in \mathbb{C}\setminus\{0\}$ and $\mathbf{k} \in \mathbb{Z}_+^2$, is to recover elements $\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_N \in [0, 2\pi)^2$ using some samples of f. Recently, such problem has received a lot of attention, and different approaches have been proposed to obtain its solution. For example, [**?**] relies on the kernel basis analysis of the multilevel Toeplitz matrix of moments of f. In [**?**], the exponential analysis has been considered as a Padé approximation problem. In contrast to the previous one, the algorithms developed in [**?**, **?**] use sampling of f along several lines in the hyperplane to obtain the univariate analog of the problem, which can be solved by classical one-dimensional approaches. Nevertheless, stability of numerical solutions in the case of noise corruption still has a lot of open questions, especially when the number of parameters increases.

Inspired by the one-dimensional approach developed in [**?**], we propose to use the method of Prony-type polynomials (the PTP method), where the parameters $\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_N$ can be recovered as a set of common zeros of the monic bivariate polynomial of an appropriate multi-degree. Numerical computations show that, even if N grows, the PTP method is more resistant to the noise than other methods. Besides, combining the PTP method with an autocorrelation sequence allows to improve the stability of the method in general.

This is joint work with Jürgen Prestin (Institute of Mathematics, University of Lübeck).

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