Dynamical Sampling on Finite Index Sets

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Dynamical Sampling aims to subsample solutions of linear dynamical systems at various times. In a real world scenario one might think of sensors on the ground measuring temperature or other quantities. Instead of installing a dense network of sensors, the idea is to use less sensors and exploit the dynamical process which the measured quantity is subject to. A natural mathematical model of this framework consists of considering inner products (the samples) of the form $\langle h, A^n f_i \rangle$, where h is the signal (temperature distribution etc.), $(f_i)_{i \in I}$ a system of fixed vectors (corresponding to the locations of the sensors), and A a linear evolution operator which is connected with the dynamical system. The objective of the general Dynamical Sampling problem is to figure out for which operators A and which families $(f_i)_{i \in I}$ the (arbitrary) signal h can be stably recovered from the sampling data $\{\langle h, A^n f_i \rangle\}_{n,i}$.

Here, we only consider finite index sets I. We start off with the finite-dimensional situation for which we prove a characterization theorem. In the infinite-dimensional case we begin by only allowing normal operators A. Here, we prove that the operator A is necessarily a diagonal operator with eigenvalues λ_j of multiplicity at most $|I|$ in the open unit disk. Moreover, the eigenvalue sequence $(\lambda_j)_{j \in \mathbb{N}}$ must be a finite union of so-called *uniformly separated* sequences. We will complete this list of necessary conditions to a characterization of the problem.

In the case where the operator A is not assumed to be normal, we also provide a (less explicit) characterization which allows for deducing necessary conditions on the spectrum of the operator A. In the case of one iterated vector, we find a more explicit characterization in terms of inner functions in the unit disk.

The talk is based on joint works with C. Cabrelli, O. Christensen, M. Hasannasab, U. Molter, and V. Paternostro.