

A Deterministic Sparse FFT for Functions with Structured Fourier Sparsity

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In [2, 3], a deterministic combinatorial Fourier algorithm for estimating the best k -term Fourier representation for a given frequency sparse signal, relying heavily on the Chinese Remainder Theorem and combinatorial concepts, was introduced. Its runtime is sublinear in the input length and scales quadratically in the sparsity k . In [1], the algorithm from [2, 3] was adapted for input functions with short frequency support, achieving a runtime that is sub-quadratic in the sparsity. A similar setting was also considered in [4], where a different deterministic sublinear Fourier algorithm employing periodizations of the signal was proposed.

Inspired by these approaches, and using ideas from [2, 3], we develop a deterministic sparse Fourier algorithm for input functions where we generalize the setting of a short frequency support to a *structured frequency support*, especially allowing a support consisting of multiple long intervals. Our method then achieves a runtime that is sublinear in the input length and scales sub-quadratically in the sparsity, which, for the class of functions considered, has so far only been achieved by randomized techniques.

References

- [1] S. Bittens. Sparse FFT for Functions with Short Frequency Support. *Dolomites Res. Notes Approx.*, 10:43–55, 2017.
- [2] M. A. Iwen. Combinatorial Sublinear-Time Fourier Algorithms. *Found. Comput. Math.*, 10(3):303–338, June 2010.
- [3] M. A. Iwen. Improved Approximation Guarantees for Sublinear-Time Fourier Algorithms. *Appl. Comput. Harmon. Anal.*, 34(1):57–82, 1 2013.
- [4] G. Plonka and K. Wannewetsch. A deterministic sparse FFT algorithm for vectors with small support. *Numer. Algorithms*, 71(4):889–905, 2016.

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