Modeling and Simulation with ODE for MSE

The exercises can be handed in during the week 27.11. – 01.12.2017!

Exercise 1 (Method of Undetermined Coefficients)
Find a particular solution to the following inhomogeneous ODEs:

(a) \( y'' + 5y' + 4y = (4x^2 + 5)e^{-3x} \)
(b) \( y'' - 16y = 5e^{4x} + 3e^x \)
(c) \( y'' + y = 2\cos^2 x \) \quad \text{Hint: Use an addition theorem first.}
(d) \( y'' - 4y' + 5y = 2e^{2x}\cos x + 3e^{3x}\sin x \)

Exercise 2 (Linear independent solutions)
Let the linear ODE of second order with constant coefficients
\[ y''(x) + ay'(x) + by(x) = 0, \quad a, b \in \mathbb{R}, \]
be given, where \( D = a^2 - 4b > 0. \)
Argue that the functions
\[ y_1(x) = \exp(-\frac{a}{2}x) \sinh(\frac{\sqrt{D}}{2}x) \quad \text{and} \quad y_2(x) = \exp(-\frac{a}{2}x) \cosh(\frac{\sqrt{D}}{2}x) \]
are linear independent solutions of the ODE and thus form a basis of solutions.
Recall that the functions \textit{Sine Hyperbolicus} (\( \sinh \)) and \textit{Cosine Hyperbolicus} (\( \cosh \)) are defined as
\[ \sinh(t) = \frac{e^t - e^{-t}}{2} \quad \text{and} \quad \cosh(t) = \frac{e^t + e^{-t}}{2}. \]

Exercise 3 (Fundamental systems)
Consider the following Fundamental systems of linear homogeneous ODEs of second order (with constant coefficients):

(a) \( a_1(x) = e^{2x}\cos(x), \quad a_2(x) = e^{2x}\sin(x) \)
(b) \( b_1(x) = e^{(2+i)x}, \quad b_2(x) = e^{(2-i)x} \)
(c) \( c_1(x) = e^{(1+2i)x}, \quad c_2(x) = e^{(1-2i)x} \)
(d) \( d_1(x) = -ie^x\cos(2x), \quad d_2(x) = ie^x\sin(2x) \)
(e) \( e_1(x) = a_1(x) + b_1(x), \quad e_2(x) = a_2(x) + b_2(x) \)

Which of these Fundamental systems belong to the same ODE (i.e. yield the same space of solutions)?

Exercise 4 (Higher order ODE)
Determine the general solution of the ODE
\[ y^{(4)}(x) - y'''(x) - 3y''(x) + 5y'(x) - 2y(x) = (x + 1)e^x. \]

Information and material related to the lecture can be found at the lecture webpage
http://www-m15.ma.tum.de/Allgemeines/ModelingSimulation