Modeling and Simulation with ODE for MSE

The exercises can be handed in during the week 20. – 24.11.2017.

Exercise 1 Which of the following differential equations has \{e^{-2x}, xe^{-2x}\} as its fundamental system?

(a) \quad y'' - 4y = 4,

(b) \quad y'' + 4y' + 4y = 4,

(c) \quad y'' - 2y' - 8y = 4.

Exercise 2 (Reduction of Order)
Solve the initial value problem
\[ x^3y'' - xy' + y = 0, \quad y(1) = 0, y'(1) = 1. \]

Guess a possibly simple solution \( y_1 \neq 0 \) and then deduce a second solution.

Exercise 3 (Euler Differential Equation)
Find two linearly independent real-valued solutions of the differential equations

(a) \quad 4x^2y'' - 3y = 0,

(b) \quad x^2y'' - 3xy' + 4y = 0,

(c) \quad x^2y'' + xy' + 9y = 0.

Use the ansatz \( y(x) = x^m \) for some undetermined exponents \( m \) as the starting point for your arguments.

Exercise 4 (An application)
We consider a serial oscillatory circuit, consisting of a power supply \( U \), a resistor \( R \), a capacitor with capacity \( C \), and an inductor with inductivity \( L \). This being a serial circuit, we have
\[
U = U_C + U_R + U_L
\]
\[
U_C = \frac{Q}{C}, \quad U_R = R \cdot I = R \cdot \dot{Q}, \quad U_L = L \cdot \ddot{I} = L \cdot \ddot{Q}.
\]
From this, we immediately obtain an ODE for the charge \( Q \) within the capacitor:
\[
L\dddot{Q} + R\dot{Q} + \frac{1}{C}Q = U.
\]

Now consider a circuit with parameters \( L = 0.05 \), \( R = 10 \) and \( C = \frac{1}{2500} \).
(a) Determine the charge $Q$ and the current $I = \dot{Q}$ when there is no outside power supply ($U = 0$) for the initial state $Q(0) = 2$ and $I(0) = 0$.

(b) Now consider an outside power source $U(t) = U_0 \cos(\omega t)$ with $U_0 = 500$ and $\omega = 100$. Determine $Q(t)$ and $I(t)$ for initial conditions $Q(0) = 0.1$, $I(0) = 20$.

(c*) For the above parameters, consider the dependence of the maximum of the charge $Q$ and current $I$ on the frequency $\omega$.

Information and material related to the lecture can be found at the lecture webpage

http://www-m15.ma.tum.de/Allgemeines/ModelingSimulation