Modeling and Simulation with ODE for MSE

The exercises can be handed in during the week 15.01. – 19.01. 2018

Exercise 1 (Parameter multistep method)

Let $a \in \mathbb{R}$ and consider the following family of multistep methods:

$$y_{n+2} + \alpha_1 y_{n+1} + a y_n = h(\beta_2 f(t_{n+2}, y_{n+2}) + \beta_1 f(t_{n+1}, y_{n+1}) + \beta_0 f(t_n, y_n)).$$

Determine $\alpha_1, \beta_0, \beta_1, \beta_2 \in \mathbb{R}$ such that the family of multistep methods has order of consistency 4, i.e.,

$$|y_{n+2} - y(x_{n+2})| = O(h^4)$$

provided all data from previous iteration steps are exact, i.e. $y(x_k) = y_k$ for all $k < n + 2$. For simplicity, one may also assume $f(t_{n+2}, y_{n+2}) = f(t_{n+2}, y(x_{n+2}))$.

Exercise 2 (Characterization of order)

Consider the multistep scheme

$$y_{n+2} - y_n = \frac{1}{3} h (f(t_{n+2}, y_{n+2}) + 4 f(t_{n+1}, y_{n+1}) + f(t_n, y_n))$$

for approximating the solution to

$$y'(t) = f(t, y(t)), y(t_0) = y_0.$$ 

Calculate the order of the scheme.

Exercise 3 (Adams-Bashforth and Adams-Moulton methods)

Try to recall the constructions of the Adams-Bashforth and Adams-Moulton multistep methods from the lecture. In particular, show the iteration formula

$$y_{n+1} = y_n + \frac{h}{12} \left( 5 f(t_{n+1}, y_{n+1}) + 8 f(t_n, y_n) - f(t_{n-1}, y_{n-1}) \right)$$

for the 2-step Adams-Moulton method. Further determine its order of consistency.

Information and material related to the lecture can be found at the lecture webpage

http://www-m15.ma.tum.de/Allgemeines/ModelingSimulation