Modeling and Simulation with ODE for MSE

The exercises should be handed in during the week 30.10. – 03.11.!

Exercise 1 (Solution?)
Check whether \( u \) is actually a solution of the ODE! Try to correct the solution suggestion if not!

(a) \( u'(t) = u(t) + e^t \) and \( u(t) = (c - t)e^t \);
(b) \( u'(t) \tan(t) = 2u(t) - 8 \) and \( u(t) = c\sin^2(t) + 4 \).

Exercise 2 (Quiz)
Which of the following plots (a),(b),(c) cannot be a solution plot of an ODE of the form \( u'(t) = f(t, u) \) with continuous right hand side \( f \)? Why?

Exercise 3 (An application)
During clinical studies A.K. Kaid determined in 1964 that the growth of tumors can be described with the help of a model proposed by B. Gompertz in 1825:

\[
y' = -ay\ln(y/y_\infty).
\]

Therein \( y(t) \) is the mass of the tumor at tim \( t \), and \( a \) and \( y_\infty \) are positive parameters.

(a) What is the domain of definition of the right-hand side?
(b) Determine units for mass and time, so that \( a = 1 \) and \( y_\infty = 1 \).
(c) Analyze the right-hand side, and determine conditions under which the tumor is growing or shrinking.
(d) Determine the general solution of the differential equation. Which interpretation admit the parameters $a$ and $y_\infty$?

(e) What is the solution with initial value $y(0) = 0$? Is it unique?

**Exercise 4** (Picard-Lindel"of iteration vs. Taylor approximation)

Consider the initial value problem (IVP) for the following differential equation

$$y'(x) = f(x, y(x)) = y^2(x) + x^2, \quad y(x_0) = y_0.$$ 

This ODE is of Ricatti-type, a comparatively simply type of nonlinear equations with non-separable variables.

(a) Explain that for every initial point $(x_0, y_0) \in \mathbb{R}^2$ the IVP is uniquely solvable.

(b) For the solution $y(x)$ of the IVP with initial value $y(0) = 0$ determine its Taylor polynomial $T_7$ of degree 7.

*Hint:* Using the ODE allows to iteratively calculate all derivatives of $y$, and consequently also all values of those derivatives in $x = 0$

(c) Again for the initial value $y(0) = 0$ calculate the first three iterates of the Picard-Lindel"of iteration, and compare those to the Taylor polynomial $T_7$.

Information and material related to the lecture can be found at the lecture webpage

http://www-m15.ma.tum.de/Allgemeines/ModelingSimulation