

Fallstudien der Mathematischen Modellbildung

Exercise Sheet 3

October 22, 2015

Problem 1. Let $\varphi \in C(\mathbb{R}) \cap L^1(\mathbb{R})$ be **not** $(-L/2, L/2)$ -bandlimited, while satisfying

1. $\{\varphi(\frac{n}{L})\}_{n \in \mathbb{Z}} \in \ell^2(\frac{\mathbb{Z}}{L})$,
2. $\mathcal{F}\varphi \in L^1(\mathbb{R})$,
3. $|\varphi(x)| + |\mathcal{F}\varphi(x)| \leq \frac{C}{(1+|x|)^{1+\varepsilon}}$ for all $x \in \mathbb{R}$, for some $C, \varepsilon > 0$.

Let $\psi : \mathbb{R} \rightarrow \mathbb{R}$ be the time-continuous function obtained by applying the Whittaker-Shannon reconstruction formula to the samples $\varphi(\frac{n}{L}), n \in \mathbb{Z}$. Compute the Fourier transform $\mathcal{F}\psi$ and explain how it is related to the Fourier transform of the original function φ .