Themenauswahl für den ersten Teil der Vorlesung Fallstudien der Mathematischen Modellbildung [MA2902]

TU München, Zentrum Mathematik
Lehrstuhl für Angewandte Numerische Analysis
WS 2015/2016

February 2, 2016

Thema 1. With this theme you have to give an accurate presentation of the Whittacker-Shannon Sampling Theorem and of its real-life applications. As a reference, you may consider [1], Chapter 8 of [2] and [3]. You will have to

1. explain in detail the steps of the proof of the Whittacker-Shannon Sampling Theorem;
2. comment the meaning of this theorem and its limitations in relationship with the Heisenberg Uncertainty Principle;
3. present the problems that may occur in case one tries to reconstruct a signal by using this result without meeting its hypothesis.

Thema 2. This theme is concerned with the issue of using frames in signal reconstruction. As material for this theme see [1], [4], [5] and [6]. You shall

1. present the theory of frames in Hilbert spaces, highlighting the most important results, in particular the importance of the redundancy with respect to the recovery from noisy coefficients of elements of the Hilbert space;
2. introduce the concept of Gabor frames in Hilbert spaces (extensively covered in [5]), and focus on Gabor frames in finite dimensions, introducing the heuristic idea behind them;
3. list their potential applications (e.g., in music analysis) with the aid of MatLab simulations.

Thema 3. This theme focuses on the Fast Fourier Transform on two-dimensional data. Suggested references for this theme are [1], [7], [8] and [9].

1. generalize the Fast Fourier Transform and Discrete Fourier Transform algorithms for 1D-vectors in $\ell^2(\mathbb{Z}_N)$ (where $N = 2^n$) to 2D-matrices in $\ell^2(\mathbb{Z}_N \times \mathbb{Z}_M)$ (where $N = 2^n$ and $M = 2^m$);

2. calculate and compare the complexity of the above algorithms;
3. test the implemented algorithms, by providing convincing reconstruction results.

References

[1] Massimo Fornasier, *Slides of the course "Fallstudien der Mathematischen Modellbildung"*, 2013. Available at
http://www-m15.ma.tum.de/Allgemeines/MatematischenModellbildung.

[2] David W. Kammler, *A first course in Fourier analysis.* Cambridge University Press, 2007.

[3] Claude E. Shannon, *Communication in the presence of noise.* Proceedings of the IRE 37.1 (1949): 10-21.

[4] Christopher Heil, *A basis theory primer.* Springer, 2011.

[5] Ole Christensen, *An introduction to frames and Riesz bases.* Springer, 2003.

[6] Hans G. Feichtinger, Franz Luef, and Tobias Werther. *A guided tour from linear algebra to the foundations of Gabor analysis.* Gabor and wavelet frames 10 (2007): 1-49.

[7] Paul Bourke, *DFT and FFT*, 1998. Available at
http://paulbourke.net/miscellaneous/dft/

[8] Juan J. Alonso, *Lecture Notes for Parallel Methods in Numerical Analysis.* Available at
http://adl.stanford.edu/cme342/Lecture_Notes_files/lecture24-05.pdf

[9] John M. Brayer, *Introduction to Fourier Transform for Image Processing.* Available at
https://www.cs.unm.edu/~brayer/vision/fourier.html