Gaussian normal equation

**Theorem 1.** Let \( y \in D(A^\dagger) \). Then \( x \in X \) is a least-squares solution of \( Ax = y \) if and only if the normal equation
\[
A^*Ax = A^*y
\]
holds.

**Proof.** An element \( x \in X \) is a least-squares solution if and only if \( Ax \) is the projection of \( y \) onto \( R(A) \), which is equivalent to \( Ax - y \in R(A)^\perp \). Since \( R(A)^\perp = N(A^*) \), this is equivalent to \( A^*(Ax - y) = 0 \). \( \square \)