Exercise Sheet 1

1. Assume $E$ and $F$ are normed spaces. Show that $L(E, F)$ becomes a Banach space provided $F$ is a Banach space.

2. Let $E$ be a finite dimensional normed space and $F$ an arbitrary normed space. Show that $E$ is a Banach space and every linear mapping $E \to F$ is continuous.

3. Suppose that $E_1, \ldots, E_n, F$ are normed vector spaces and $f : \prod_{i=1}^{n} E_i \to F$ multilinear. Show that the following statements are equivalent.
   a) $f$ is continuous on $\prod_{i=1}^{n} E_i$.
   b) $f$ is continuous at $0 \in \prod_{i=1}^{n} E_i$.
   c) $\|f(x_1, \ldots, x_n)\|$ is bounded on the product of the unit balls $\|x_1\| \leq 1, \ldots, \|x_n\| \leq 1$.

4. Let $\| \cdot \| : L(\prod_{i=1}^{n} E_i, F) \to \mathbb{R}_{0}^{+}$ be defined by
   $$\|f\| := \sup \{ \|f(x_1, \ldots, x_n)\|_F : \|x_i\|_{E_i} = 1, \forall i \in \{1, \ldots, n\} \}.$$
   Show that $\| \cdot \|$ defines a norm on $L(\prod_{i=1}^{n} E_i, F)$. Moreover, $L(\prod_{i=1}^{n} E_i, F)$ is a Banach space whenever $F$ is one.